1 Introduction

Some recent papers consider taxing robots to redistribute income to these displaced workers.[1] Implementing taxes such as a "robot tax" is challenging, in part because of the difficulty in clearly separating which intermediate goods perfectly substitute for labor from those which complement it. Also, tax avoidance will be a concern.[2] To address these practical issues, this note analyzes the welfare consequences of imposing a tax on intermediate goods when their type cannot be determined by the planner with a restricted production functional form.

In particular, my model considers a two-by-two scenario: two types of labor (routine vs. non-routine) and two types of intermediate goods (*displacing intermediate goods*—e.g., self-check-out cash registers and self-driving trucks—, which are complements to non-routine labor but are perfect substitutes for routine labor, and *complementary intermediate goods*—e.g., conventional cash registers and conventional trucks—, which are more complementary to routine labor than to non-routine labor). In my model, complementary intermediate goods and routine labor jointly work as one input that is perfectly substitutable to displacing intermediate goods. These two inputs produce an intermediate output that is complementary to non-routine labor. In this short note, I focus on a specialized case in which routine labor is a perfect complement to complementary intermediate goods, while the final output is produced under a Cobb-Douglas production function whose inputs are the intermediate output and non-routine labor. With this functional form assumption, a uniform tax rate over different intermediate good types is assumed to address the aforementioned screening and tax avoidance concerns. Moreover, following Guesnerie (1998), I focus on a proportional intermediate good tax to remove arbitrage opportunities in the resale market.

Due to asymmetric information on intermediate good types, there are two opposing forces of non-discriminatory intermediate good taxation in welfare. On the one hand, by taxing *only* displacing intermediate goods, the planner can reduce the wage gap between the two labor types. Taxing a complement of non-routine workers will decrease non-routine-worker wage rates while taxing a substitute for routine workers will increase routine-worker wage rates. The reduction of the wage gap will relax the incentive compatibility constraint of non-routine workers to mimic routine workers—that is, a reduction in the information rent of non-routine types—in the planner's welfare maximization program, as studied in Naito (1999). On the other hand, taxing *only* complementary intermediate goods will decrease the wage rates of not only non-routine workers but also routine workers since complementary intermediate goods complement both types of labor. A decrease in the routine-workers' wage rates may decrease welfare and possibly dominate the positive redistribution effects from taxing displacing intermediate goods.

Despite these competing forces and complex settings, I find that there is a simple solution in which on top of optimal income taxation, the planner imposes a strictly positive proportional intermediate good tax that is non-discriminatory over types of intermediate goods, for redistributive purposes. There are two key driving forces for this result.

One is the relative labor complementarity of intermediate goods. While displacing intermediate goods are perfect substitutes for routine labor and complementary to non-routine

[1] These papers are Costinot and Werning (2022), Thuemmel (2022), and Guerreiro et al. (2022).

[2] See, e.g., Slemrod and Kopczuk (2002).

labor, complementary intermediate goods are complements to *both* routine and non-routine labor. This implies that while taxing displacing intermediate goods—i.e., a positive force on the marginal productivity of routine labor and a negative force on that of non-routine labor—will unambiguously reduce the wage differential, taxing complementary intermediate goods may or may not widen the wage gap. This is because taxing complementary intermediate goods will decrease not only the wage rate of routine labor, but also that of non-routine labor. Thus, even though the wage gap may increase through taxing complementary intermediate goods, the magnitude is not as much as the reduced gap by taxing displacing intermediate goods. Then, the difference in relative labor complementarity tends to push the sign of the optimal intermediate good tax to a positive direction.

To explicitly depict how the model works, I work on a separate model in which the planner can perfectly distinguish between two types of intermediate goods. In this setting, the planner imposes differential tax rates on different types of intermediate goods. I show that the optimal tax regime involves a positive tax rate on displacing intermediate goods and a negative tax rate on complementary intermediate goods. In particular, I demonstrate that as long as the resource and incentive compatibility constraints hold, the tax regime features a full subsidy on complementary intermediate goods. This is because of complementarity between complementary intermediate goods and both routine and non-routine labor which cushions production loss and because of the production functional form.

While the main results are obtained based upon the functional form assumption about the perfect substitutability between the joint input and displacing intermediate goods, it is technically difficult to relax this assumption. As soon as it departs from perfect substitution, the model becomes analytically intractable. Meanwhile, given that using the results from Acemoglu and Restrepo (2020), Thuemmel (2022) calibrates his model and finds a high substitution elasticity at 4.41 between robots and labor, the perfect substitution assumption may not be too far from real-life applications.

The other force driving the positive result exploits the price difference between the two types of intermediate goods. As a unique feature of an automation model, perfectly substituting intermediate goods such as robots need zero routine labor by definition. This implies that the price of complementary intermediate goods has to be lower than that of displacing intermediate goods in a partially automated economy since complementary intermediate goods require routine labor for potentially automatable tasks. Thus, if we impose a uniform proportional tax on both types of intermediate goods, the tax burden is placed disproportionally on displacing intermediate goods than complementary intermediate goods since displacing intermediate goods are more expensive. This differential tax burden also reduces the wage gap between the two worker types.

The closest paper to this short note is Slavík and Yazici (2014). They have two different types of capital in their dynamic model: equipment and structure. Equipment capital is more complementary to high-skilled labor than low-skilled labor, so that an increase in equipment capital widens the wage gap. Structure capital is neutral in a sense that a change in structure capital amounts will not affect the wage ratio. While their quantitative exercise covers richer scenarios, in their theoretical part, they assume that different tax rates can be imposed over the two different capital types. Focusing on a special production functional form, this note relaxes this assumption in a steady-state case.

2 Environment

2.1 Household

Suppose a continuum of households with a unit measure. These households are decomposed to π_j households for $j \in \{n, r\}$ where subscript n and r denote the non-routine and routine labor types, respectively. A household of type j's optimization problem is

$$\begin{aligned} &\underset{c_j, l_j}{\text{maximize}}\; U_j = u(c_j, l_j) \\ &\text{subject to } c_j \leq w_j l_j - T(w_j l_j), \end{aligned}$$

where c_j denotes consumption amounts, l_j labor amounts, w_j wage rates, and $T(.)$ the income tax schedule. Note that the price of a single consumption good is normalized to 1.

For convenience, write $u_x = \partial u(c, l)/\partial x$ where $x = c, l$ and $u_{xy} = \partial^2 u(c, l)/\partial x \partial y$. I make the standard concavity and convexity assumptions that, $u_c > 0, u_l < 0, u_{cc}, u_{ll} < 0$, and that consumption and leisure are both normal goods. Additionally, I assume that $u(c, l)$ satisfies the standard Inada conditions for interior solutions.

2.2 Intermediate Good Producers

Both types of intermediate goods are produced by perfectly-competitive intermediate good producers in the external (global) market. The marginal cost for displacing intermediate goods is ϕ_d units of output, while that for complementary intermediate goods is ϕ_c units of output.

2.3 Final good producer

The final good producer employs non-routine workers (N_n), routine labor (N_r), and buys displacing intermediate goods (X_d) and complementary intermediate goods (X_c). The production function is given as

$$Y = A\left(X_d + \min[N_r, X_c]\right)^{1-\alpha} N_n^{\alpha}$$

Note that (a) my production function contains the one from Autor et al. (2003) and the static model of Guerreiro et al. (2022) as a special case, and (b) I restrict attention to the worst possible case by considering the perfect complement relationship between routine labor and complementary intermediate goods in which the negative effect of an intermediate good tax is the highest. The profit function is:

$$Y - w_n N_n - w_r N_r - (1 + \tau_x)\left(\phi_d X_d + \phi_c X_c\right),$$

where τ_x is an ad-valorem uniform tax rate on intermediate goods.

2.4 Equilibrium

An equilibrium is defined as the collection of a set of allocations $\{c_r, l_r, c_n, l_n, N_r, N_m, X_d, X_c\}$, prices $\{w_r, w_n\}$, and a tax system $\{T(.), \tau_x\}$ such that: (i) given prices and taxes, allocations solve the households' problem; (ii) given prices and taxes, allocations solve the firms' problem; and (iii) markets clear.

Note that the resource constraint can be written as:

$$\pi_r c_r + \pi_n c_n \leq Y - (\phi_d X_d + \phi_c X_c).$$

I assume an interior solution since corner solutions result in degenerate cases of zero intermediate good tax rates. At an interior solution, due to the perfect substitution relationship, the following condition has to hold:

$$w_r + (1+\tau_x)\phi_c = (1+\tau_x)\phi_d,$$

which implies,

$$w_r = (1+\tau_x)(\phi_d - \phi_c). \tag{1}$$

With $N_r = X_c$, one can still take the first-order conditions of the final good producer's profit function with respect to X_d and N_n and get

$$w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[(1+\tau_x)\phi_d]^{\frac{1-\alpha}{\alpha}}}, \tag{2}$$

The wage rate of non-routine labor does not depend on ϕ_c due to the perfect substitutability between displacing intermediate goods and the joint input of routine labor and complementary intermediate goods. Note that the constant returns to scale production function induces a linear production possibility frontier, leading to a fixed proportion of factor demands to satisfy the zero profit condition. In a conventional model with factors that only have imperfectly substitutable labor types, the factor ratio cannot be solved explicitly with exogenous parameters and equilibrium wages are expressed as the marginal products of factors that are endogenous functions of the factor ratio. In my automation model, this ratio can be solved explicitly as a function of exogenous parameters for the following reason.

First, the perfect substitutability relationship allows the marginal product of non-routine labor to depend upon only the ratio between non-routine labor amount and $y = X_d + \min[N_r, X_c]$. Second, the interior solution assumption ensures equation (1) and thus the cost of y is the cost of displacing intermediate goods that is exogenously given. Then, the ratio of non-routine labor to y can be determined by the exogenous parameters, and thus the *equilibrium* non-routine labor wage can be expressed as the exogenous parameters. On the other hand, the equilibrium routine labor wage directly comes from the interior solution assumption. Therefore, the equilibrium factor ratios are chosen such that equation (1) and (2) are both satisfied.

3 Result

3.1 Asymmetric Information at Both Household And Production Sides

By the revelation principle, the policymaker's problem comes down to choosing allocations $\{c_j, l_j\}_{j=r,n}$ and a uniform intermediate good tax rate τ_x to maximize the utilitarian social welfare $\pi_r U_r + \pi_n U_n$. Fix τ_x first and assume that the policy maker has chosen optimal income tax level $\{c_j, l_j\}_{j=r,n}$ that are functions of τ_x. Define $W(\tau_x) = \max \pi_r u(c_r, l_r) + \pi_n u(c_n, l_n)$. Then, the remaining social planner's problem is

$$
\begin{aligned}
W(\tau_x) := \max_{c_r, l_r, c_n, l_n} \ & \pi_r u(c_r, l_r) + \pi_n u(c_n, l_n) \\
& \text{subject to} \\
& [\eta_r \pi_r] \quad u(c_r, l_r) \geq u\left(c_n, \frac{w_n}{w_r} l_n\right), \\
& [\eta_n \pi_n] \quad u(c_n, l_n) \geq u\left(c_r, \frac{w_r}{w_n} l_r\right), \\
& [\mu] \quad \pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \left(\frac{\tau_x + \alpha}{\alpha(1+\tau_x)}\right) + \frac{\pi_r w_r l_r}{(1+\tau_x)}
\end{aligned}
$$

where elements in the closed parentheses are the Lagrangian multipliers, the first constraint the incentive compatibility constraint for routine households, the second that for non-routine households, and the third resource constraint after plugging in the equilibrium conditions including $\pi_r l_r = X_c$. I focus on cases where the incentive compatibility constraint for non-routine workers binds, while that for routine workers slacks, which Stiglitz (1982) calls a *normal case.*

Proposition 1. *Assume $\eta_r = 0$ and $\eta_n > 0$. Suppose the uniform capital tax rate is initially non-positive including zero. Then, at an interior solution, the optimal intermediate good tax is strictly positive.*

Proof. Letting $\eta_r = 0$, by envelope theorem, we get,

$$
\begin{aligned}
W'(\tau_x) = & -\eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_n} l_r\right) \frac{d \log (w_r/w_n)}{d \log (1+\tau_x)} \frac{1}{1+\tau_x} \frac{w_r l_r}{w_n} \\
& + \mu \Bigg[\pi_n w_n l_n \frac{\tau_x + \alpha}{\alpha(1+\tau_x)^2} \left(\frac{d \log w_n}{d \log(1+\tau_x)} + \frac{1-\alpha}{(\tau_x+\alpha)}\right) + \\
& \qquad \frac{\pi_r w_r l_r}{(1+\tau_x)^2} \left(\frac{d \log w_r}{d \log(1+\tau_x)} - 1\right)\Bigg].
\end{aligned}
$$

Using the equilibrium wage formulas and their logarithmic derivatives, we get:

$$
W'(\tau_x) = \frac{1}{\alpha(1+\tau_x)} \left[-\eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_n} l_r\right) \frac{w_r l_r}{w_n} - \mu \pi_n w_n l_n \frac{\tau_x}{1+\tau_x} \frac{1-\alpha}{\alpha}\right]
$$

Since $\mu, \eta_n > 0$, if $\tau_x \leq 0$, we get:

$$W'(\tau_x) > 0.$$

This implies that at zero intermediate-good tax rate, the planner can increase welfare by marginally increasing the tax rate. ■

3.2 Second-best: No Asymmetric Information over Intermediate Good Types

In this section, I analyze a situation where the government cannot perfectly discriminate between two types of labor but can perfectly distinguish between two types of capital. Denote by τ_d as the capital tax on displacing capital and by τ_c as that on complementary capital. Both taxes are bounded from below but not above: $-1 \leq \tau_d < \infty$ and $-1 \leq \tau_r < \infty$.

Note that the wage rates in this case become

$$w_r = (1+\tau_d)\phi_d - (1+\tau_c)\phi_c.$$

$$w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[(1+\tau_d)\,\phi_d\right]^{\frac{1-\alpha}{\alpha}}},$$

With this, in general, I obtain a result where the optimal capital tax rate for displacing capital is positive and that for complementary capital is negative. In particular, the wage function for routine labor under the perfect complementarity implies a linear gain in the wage rate of routine labor through a tax subsidy on complementary intermediate goods. The envelope theorem reveals that a marginal change in the tax rate of complementary intermediate goods does not affect the resource constraint at a first-order scale, partly due to complementarity between complementary intermediate goods and both routine and non-routine labor which cushions production loss, and partly due to the production functional form. It turns out that the linear gain *always* dominates the second-order production loss at any tax rate τ_c, and thus $\tau_c = -1$ at an optimum level as long as both IC constraints for non-routine labor and resource constraint bind and are not violated at any subsidy level.

Nonetheless, the main result implies that a marginal increase in the tax rate of displacing intermediate goods dominates whenever the production loss is small enough. This is because the wage gap reduction is larger through a marginal increase in τ_d than a decrease in τ_c as ϕ_c and thus τ_c do not appear on the wage function of non-routine labor due to the perfect substitutability relationship while τ_d has a positive linear effect on w_r and has a negative nonlinear effect on w_n.

Proposition 2. *Assume $\eta_r = 0$ and $\eta_n > 0$. Then, at an interior solution, the optimal tax for displacing capital is always strictly positive ($\tau_d^\star > 0$) if the tax rate is initially non-positive, while that for complementary capital is always strictly negative ($\tau_c^\star < 0$) and features a full subsidy $\tau_c^\star = -1$ at an optimum level as long as both IC constraints for non-routine labor and resource constraint bind and are not violated at any subsidy level.*

Proof. Let $\tau_x = (\tau_d, \tau_c)$. Then, the social planner's optimization problem is

$$
\begin{aligned}
W(\tau_x) := \max_{c_r, l_r, c_n, l_n} \; & \pi_r u(c_r, l_r) + \pi_n u(c_n, l_n) \\
& \text{subject to} \\
& [\eta_r \pi_r] \quad u(c_r, l_r) \geq u\left(c_n, \frac{w_n}{w_r} l_n\right), \\
& [\eta_n \pi_n] \quad u(c_n, l_n) \geq u\left(c_r, \frac{w_r}{w_n} l_r\right), \\
& [\mu] \quad \pi_r c_r + \pi_n c_n + G \leq \pi_n w_n l_n \left(\frac{\tau_d + \alpha}{\alpha(1+\tau_d)}\right) + \pi_r w_r l_r \frac{\phi_d - \phi_c}{(1+\tau_d)\phi_d - (1+\tau_c)\phi_c}.
\end{aligned}
$$

Letting $\eta_r = 0$, and taking the partial derivative of the welfare function with respect to τ_c, we get

$$
\frac{\partial W(\tau_c)}{\partial \tau_c} = - \eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_n} l_r\right) \frac{dw_r}{d\tau_c} \frac{l_r}{w_n}. \tag{3}
$$

With the equilibrium wages we have gotten above, we get:

$$
w_r = (1+\tau_d)\phi_d - (1+\tau_c)\phi_c \Rightarrow \frac{dw_r}{d\tau_r} = -\phi_c
$$

Plugging this back into the envelope condition above, we get:

$$
\frac{\partial W(\tau_c)}{\partial \tau_c} = \eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_r} l_r\right) \frac{l_r}{w_n} \phi_c \quad .
$$

Notice that since $\eta_n > 0$, this term is negative. This holds true at all τ_c. Therefore, we get

$$
\frac{\partial W(\tau_c)}{\partial(-\tau_c)} > 0. \tag{4}
$$

Therefore, the social planner can achieve greater welfare by imposing a positive complementary capital subsidy, $\tau_c < 0$.

Furthermore, the condition above implies that the optimal $\tau_c = -1$ as long as both IC constraints for non-routine labor and resource constraint bind and are not violated at any subsidy level. The intuition behind this result is discussed in the main text.

Next, taking the partial derivative of the welfare function with respect to τ_d, we get

$$
\begin{aligned}
\frac{\partial W(\tau_d)}{\partial \tau_d} = & - \eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_r} l_r\right) \frac{d\log(w_r/w_n)}{d\log(1+\tau_d)} \frac{1}{1+\tau_d} \frac{w_r l_r}{w_n} \\
& + \mu \left[\pi_n w_n l_n \frac{\tau_d + \alpha}{\alpha(1+\tau_d)^2} \left(\frac{d\log\left[w_n\left(\frac{\tau_d+\alpha}{\alpha(1+\tau_d)}\right)\right]}{d\log(1+\tau_d)}\right)\right].
\end{aligned} \tag{5}
$$

With the equilibrium wages we have gotten above, we get:

$$w_r = (1+\tau_d)\phi_d - (1+\tau_c)\phi_c \Rightarrow \frac{d\log w_r}{d\log(1+\tau_d)} = \frac{\phi_d(1+\tau_d)}{(1+\tau_d)\phi_d - (1+\tau_c)\phi_c}$$

$$w_n = \alpha\frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[(1+\tau_d)\,\phi_d]^{\frac{1-\alpha}{\alpha}}} \Rightarrow \frac{d\log w_n}{d\log(1+\tau_r)} = -\frac{1-\alpha}{\alpha}$$

$$\frac{w_r}{w_n} = \frac{(1+\tau_d)\phi_d - (1+\tau_c)\phi_c}{\alpha\frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[(1+\tau_d)\phi_d]^{\frac{1-\alpha}{\alpha}}}} \Rightarrow \frac{d\log w_r/w_n}{d\log(1+\tau_r)} = \frac{\phi_d(1+\tau_d)}{(1+\tau_d)\phi_d - (1+\tau_c)\phi_c} + \frac{1-\alpha}{\alpha}$$

Plugging these back into the envelope condition above, we get:

$$\frac{\partial W(\tau_d)}{\partial \tau_d} = -\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_r}l_r\right)\frac{w_r l_r}{w_n}\left(\frac{\phi_d}{(1+\tau_d)\phi_d - (1+\tau_c)\phi_c} + \frac{1-\alpha}{\alpha(1+\tau_d)}\right) +$$
$$\mu\left(\pi_n w_n l_n \frac{\tau_d+\alpha}{\alpha\,(1+\tau_d)^2}\left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_d+\alpha}\right]\right).$$

Given that the optimal $\tau_c = -1$, we get

$$\frac{\partial W(\tau_d)}{\partial \tau_d} = -\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_r}l_r\right)\frac{w_r l_r}{w_n}\frac{1}{\alpha(1+\tau_d)} +$$
$$\mu\left(\pi_n w_n l_n \frac{\tau_d+\alpha}{\alpha\,(1+\tau_d)^2}\left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_d+\alpha}\right]\right).$$

Notice that the first is always positive. Note that the second term is non-negative when

$$\frac{\tau_d+\alpha}{\alpha\,(1+\tau_d)^2}\left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_d+\alpha}\right] \geq 0$$

This is true when $\tau_d \leq 0$. Given this condition, we get

$$\frac{\partial W(\tau_d)}{\partial \tau_d} > 0.$$

Thus, the social planner can improve welfare by marginally increasing τ_d if there has not been a positive tax rate on displacing intermediate goods yet. ■

4 Conclusion

This note finds that despite the asymmetric information problems, the optimal uniform intermediate good tax rate over different types of intermediate goods is strictly positive with the production functional form assumptions, as long as the solution is interior. The author is currently exploring on a project related to this note in the direction of characterizing the optimal marginal income tax rates under automated societies (Koizumi (2022)).

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